While you are implementing intermediate code generation and x86 machine code generation in your Roost compilers, we will begin exploring machine-independent optimization of code. Automatically improving the efficiency of code is a tall order, especially considering it must not break the code (change what it computes) along the way! We will spend 2+ weeks on a survey of optimization topics. We start with small, ad-hoc optimizations. Next week, we will show how a fairly general theoretical model can support clean and principled implementations of many sophisticated analyses and optimizations necessary to generate efficient code.

The readings and exercises are in parts: control-flow graphs and an introduction to optimization; local optimizations; and data-flow analysis. This set of reading and exercises is a little larger than usual, since we did not have meetings this past week. We will preview some parts during class on Friday.

- The first set of readings/exercises gives an overview of control-flow graphs and optimization in general.
- The second set of readings and exercises explores simple, local optimizations for eliminating redundant expressions with a technique called value numbering. This optimization is local in that it considers only single basic blocks – small stretches of code that always execute together in all executions of the program.
- The third set of readings and exercises introduces the idea of data-flow analysis to support a few specific optimizations that reason about the flow of data through more complicated control flow structures like conditionals and loops. Data-flow analyses in general compute facts or invariants that must be true at the beginning and end of each basic block in a Control Flow Graph for a single method/function/procedure body.

Next week we will continue with more depth on data-flow analysis and other techniques.

2 Readings

Control-flow graphs, basic blocks, and optimization:

- Dragon 8.4
  Alternative: EC 5.2, 5.3.2
- EC 8.1 - 8.3 (Skip or skim 8.2.1)

Local optimizations:

- EC 8.4 (Skip or skim 8.4.2)
- Dragon 8.5

Intro to data-flow analysis:

- Dragon 9 – 9.2
3 Exercises

1. Consider the following TAC code:

```plaintext
x = 2
y = 3
z = 11
L0:
    T0 = x < 10
    fjump T0 L1
T1 = x < y
    fjump T1 L3
T2 = x + 1
x = T2
    jump L2
L2:
    T3 = y < 100
    fjump T3 L2
T4 = y + 1
y = T4
    jump L3
L3:
    T5 = z + 3
z = T5
    jump L0
L1:
```

(a) Build the basic blocks and control flow graph for this code.
(b) Identify the natural loops.

2. Consider the following two basic blocks:

```plaintext
a = b + c  
\[ \text{d} = c \]  
\[ \text{e} = c + c \]  
\[ \text{f} = a + d \]  
\[ \text{g} = b + e \]  
\[ \text{h} = b + c \]
```

(a) Build a DAG for each block to show the dependences between the operations it performs. (Dragon and EC use different DAG notation. The Dragon book form is more flexible.)
(b) Perform local value numbering separately on each of the two basic blocks.
(c) Explain any differences in the redundancies found by these two techniques.
(d) At the end of each block, \( f \) and \( g \) have the same value. Why do the algorithms have difficulty discovering this fact?

3. (Dragon 8.5.6) Consider this basic block of intermediate code that uses C-style pointers. Recall that the expression \( *p \) uses the value of \( p \) as an address, referring to the contents of the memory location given by that address (not to the contents of variable \( p \) itself).

```plaintext
a[i] = b
*p = c
d = a[j]
e = *p
*p = a[i]
```
(a) Assume \( p \)'s value is unrestricted. In other words, \( p \) may hold the address of (i.e., point to) any location in memory. Construct the DAG for the basic block.

(b) Assume \( p \)'s value is restricted to hold the address of (i.e., point to) only the storage for \( b \) or \( d \). Construct the DAG for the basic block.

4. For the Control Flow Graph (CFG) in Dragon Figure 9.10 (attached):

(a) Identify the loops.

(b) Statements (1) and (2) in \( B_1 \) are both copy statements, in which \( a \) and \( b \) are given constant values. For which uses of \( a \) and \( b \) can we perform copy propagation and replace these uses of variables by uses of a constant. Do so, wherever possible, and show the resulting CFG. Do any statements become Dead Code?

(c) Identify any global common subexpressions for each loop, and eliminate them wherever possible. Show the resulting CFG.

(d) Using the CFG from (b), Identify any induction variables for each loop. Be sure to take into account any constants introduced in (b). Can strength reduction and/or induction variable elimination be applied? If so, show the resulting CFG. If not, describe one or two instructions that, if added to the flow graph, would result in an opportunity for strength reduction.

(e) Does the CFG from part (c) contain any loop-invariant computations to which code motion can be applied? If not, describe one or two instructions that, if added to the flow graph, would result in an opportunity for code motion.

5. For the CFG in Dragon Figure 9.10 (attached), compute the following:

(a) Reaching Definitions:
   - The \( gen \) and \( kill \) sets for each block. I usually represent this information in a table like the one started below:

<table>
<thead>
<tr>
<th>Block</th>
<th>( gen )</th>
<th>( kill )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_1 )</td>
<td>(1), (2)</td>
<td>(8), (10), (11)</td>
</tr>
<tr>
<td>( B_2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( B_3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( B_4 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( B_5 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( B_6 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   - The \( in \) and \( out \) sets for each block. Write the \( in \) and \( out \) sets on the CFG (attached) while working through the algorithm. Print a few copies to use for the other parts of this problem.

(b) Available Expressions:
   - The \( e_{\text{gen}} \) and \( e_{\text{kill}} \) sets for each block. For the \( e_{\text{kill}} \) sets, you may use a description like "all expressions using \( a \) or \( b \) as an operand."

   - The \( in \) and \( out \) sets for each block.

(c) Live Variables:
   - The \( def \) and \( use \) sets for each block.
   - The \( in \) and \( out \) sets for each block.

6. (Dragon 9.2.6) Prove by induction that the \( in \) and \( out \) sets for a block never shrink while computing Reaching Definitions with Dragon Algorithm 9.11 (page 607). In other words, show that, once a definition has been added to one of these sets on an iteration, all future iterations also include it in that set. Use induction over the number of iterations of the innermost loop. Consider what Reaching Definitions means and explain intuitively why we would want the proven property to hold. A succinct, careful explanation here can supplant a formal proof if needed.

7. Consider Dragon 9.2.7 or 9.2.8 and try to develop intuition for how to relate data-flow facts back to program behavior. What do they \textit{really mean}? You do not need to build full solutions.
1. \( a = 1 \)
2. \( b = 2 \)
3. \( c = a + b \)
4. \( d = c - 1 \)
5. \( d = b + d \)
6. \( d = a + b \)
7. \( e = e + 1 \)
8. \( b = a + b \)
9. \( e = c - a \)
10. \( a = b \cdot d \)
11. \( b = a - d \)