Eigenfaces for recognition (Turk & Pentland)
Principal Components Analysis (PCA)

Goal: reduce the dimensionality of the data while retaining as much information as possible in the original dataset

PCA allows us to compute a linear transformation that maps data from a high dimensional space to a lower dimensional subspace

\[
\text{covariance matrix} = \begin{bmatrix}
\text{var}(X) & \text{cov}(Y,X) \\
\text{cov}(X,Y) & \text{var}(Y)
\end{bmatrix}
\]

\[
(x,y) = w_1^*E_1 + w_2^*E_2
\]

eigenvectors of the covariance matrix are the principal components of the data

- orthogonal
- form a basis set
- eigenvector associated with largest eigenvalue is 1st principal component
Typical sample training set...

one or more images per person
aligned & cropped to common pose, size
simple background

Sample images from the Yale face database

Prepare image data for PCA:
• For each image in dataset, place columns end-to-end to create one long column vector
• Place column vectors for each image side-by-side in an MxN matrix
• Subtract the mean vector (average face) from each column
image data matrix

"eigenvectors of covariance matrix"

first principal component "eigenface"

principal components

M pixels in image

F(x,y)

E₁(x,y)

principal components "eigenfaces"

Each image in the dataset has unique set of weights

F(x,y) = Ψ(x,y) + Σᵢ wᵢ * Eᵢ(x,y)

w₁ w₂ w₃ ... wₖ
Eigenfaces for recognition (Turk & Pentland)

Perform PCA on a large set of training images, to create a set of eigenfaces, $E_i(x,y)$, that span the dataset.

First components capture most of the variation across the dataset, later components capture subtle variations.

Ψ(x,y): average face (across all faces)

Each face image $F(x,y)$ can be expressed as a weighted combination of the eigenfaces $E_i(x,y)$:

$$F(x,y) = \Psi(x,y) + \sum_i w_i \cdot E_i(x,y)$$

http://vismod.media.mit.edu/vismod/demos/facerec/basic.html

Representing individual faces

Each face image $F(x,y)$ can be expressed as a weighted combination of the eigenfaces $E_i(x,y)$:

$$F(x,y) = \Psi(x,y) + \sum_i w_i \cdot E_i(x,y)$$

Recognition process:

1. Compute weights $w_i$ for novel face image.
2. Find image $m$ in face database with most similar weights, e.g.,

$$\min \sum_{i=1}^{k} (w_i - w_i^m)^2$$