Aperture Problem

“local” motion detectors only measure component of motion perpendicular to moving edge

velocity field

velocity space

$V_y$

$V_x$

Computing 2D velocity from motion components

(1) how do we measure the perpendicular motion?

(2) how do we express these constraints on velocity?

velocity space

“intersection of constraints”

$V_y$

$V_x$

Measuring motion in one dimension

$I(x)$

$V_x$

$V_x = \text{velocity in x direction}$

- rightward movement: $V_x > 0$
- leftward movement: $V_x < 0$

- speed: $|V_x|$

- pixels/time step $V_x = \frac{\partial I}{\partial t} \frac{1}{\partial I/\partial x}$

Measuring motion components in 2-D

(1) gradient of image intensity $\nabla I = (\partial I/\partial x, \partial I/\partial y)$

(2) time derivative $\partial I/\partial t$

(3) velocity along gradient: $v^\perp$

- movement in direction of gradient: $v^\perp > 0$

- movement opposite direction of gradient: $v^\perp < 0$

$v^\perp = -\frac{\partial I/\partial t}{|\nabla I|} = -\frac{\partial I/\partial t}{[\partial I/\partial x]^2 + (\partial I/\partial y)^2]^{1/2}$
Direction of the gradient
\( \nabla I = (\partial I/\partial x, \partial I/\partial y) \)

2-D velocities \((V_x, V_y)\) consistent with \(v^\perp\)

All \((V_x, V_y)\) such that the component of \((V_x, V_y)\) in the direction of the gradient is \(v^\perp\)

\((u_x, u_y)\): unit vector in direction of gradient

Use the dot product:
\[ (V_x, V_y) \cdot (u_x, u_y) = v^\perp \]
\[ V_x u_x + V_y u_y = v^\perp \]

Time-out exercise

Details...

For each component:
1. \(u_x\)
2. \(u_y\)
3. \(v^\perp\)
4. \(V_x u_x + V_y u_y = v^\perp\)

solve for \(V_x, V_y\)
In practice...

Previously...

\[ V_x u_x + V_y u_y = v \]

New strategy:

Find \((V_x, V_y)\) that **best fits**

all motion components together

Find \((V_x, V_y)\) that minimizes:

\[ \sum (V_x u_x + V_y u_y - v^\perp)^2 \]

\[
\begin{align*}
V_x & = \int_0^1 \langle f(s, y) \rangle_x \, ds \\
V_y & = \int_0^1 \langle f(s, y) \rangle_y \, ds
\end{align*}
\]

Option 2: **Smoothness assumption:**

Compute a velocity field that:

1. is consistent with local measurements of image motion (perpendicular components)
2. has the *least amount of variation* possible

\[
\begin{align*}
\text{Pure Translation:} \\
&\begin{cases}
V_x = 0, & V_y \\
V_x, & V_y = 0
\end{cases}
\end{align*}
\]

When is the **smoothest** velocity field correct?

When is it wrong?

**motion illusions**

Computing the smoothest velocity field

motion components:

\[ V_x u_x + V_y u_y = v^\perp \]

change in velocity:

\[ (V_x^i - V_x^i+1, V_y^i - V_y^i+1) \]

Find \((V_x^i, V_y^i)\) that minimize:

\[
\sum (V_x^i u_x^i + V_y^i u_y^i - v^\perp)^2 + \lambda \left[ (V_x^i - V_x^i+1)^2 + (V_y^i - V_y^i+1)^2 \right]
\]

deviation from image motion measurements + variation in velocity field
Two-stage motion measurement

motion components → 2D image motion

Movshon, Adelson, Gizzi & Newsome

V1: high % of cells selective for direction of motion (especially in layer that projects to MT)
MT: high % of cells selective for direction and speed of motion
lesions in MT → behavioral deficits in motion tasks

Testing with sine-wave “plaids”

Moving plaid demo:
http://www.georgemather.com/MotionDemos/PlaidMP4.html

Movshon et al. recorded responses of neurons in area MT to moving plaids with different component gratings

Logic behind the experiments

Component cells measure perpendicular components of motion
e.g. selective for vertical features moving right
predicted responses: (1) yes (2) yes (3) no

Pattern cells integrate motion components
 e.g. selective for rightward motion of pattern
predicted responses: (1) no (2) no (3) yes

Movshon et al. observations

• Cortical area V1:
  all neurons behaved like component cells

• Cortical area MT:
  layers 4 & 6: component cells
  layers 2, 3, 5: pattern cells

  Evidence for two-stage motion measurement!

• Perceptually, two components are not integrated if:
  large difference in spatial frequency
  large difference in speed
  components have different stereo disparity